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An analysis of $B \to \eta' K$ decays using a global fit in QCD factorization

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Abstract. In the framework of QCD factorization, we study $B^{+(0)} \to \eta' K^{+(0)}$ decays. In order to more reliably determine the phenomenological parameters $X_{\rm H}$ and $X_{\rm A}$ arising from end-point divergences in the hard spectator scattering and weak annihilation contributions, we use the global analysis for twelve $B \to PP$ and VP decay modes, such as $B \to \pi\pi$, πK, $\rho\pi$, ρK, et cetera, but excluding the modes whose (dominant) internal quark-level process is $b \to s\bar{s}s$. Based on the global analysis, we critically investigate possible magnitudes of $X_{\rm H,A}$ and find that both large and small $X_{\rm H,A}$ terms are allowed by the global fit. In the case of the large $X_{\rm H,A}$ effects, the standard model (SM) prediction of the branching ratios (BRs) for $B^{+(0)} \to \eta' K^{+(0)}$ is large and well consistent with the experimental results. In contrast, in the case of the small $X_{\rm H,A}$ effects, the SM prediction for these BRs is smaller than the experimental data. Motivated by the recent Belle measurement of $\sin(2\phi_1)$ through $B^0 \to \phi K_s$, if we take into account possible new physics effects on the quark-level process $b \to s\bar{s}s$, we can explicitly show that these large BRs can be understood even in the small $X_{\rm H,A}$ case. Specifically, we present two new physics scenarios: R-parity violating SUSY and R-parity conserving SUSY.

1 Introduction

From B factory experiments such as Belle and BaBar, copious experimental data on B decays start to provide new bounds on previously known observables with great precision as well as an opportunity to see very rare decay modes for the first time. There exist plenty of experimental data observed for charmless hadronic decays $B \to PP$ (P denotes a pseudoscalar meson), such as $B \to \pi\pi$, πK, et cetera, and $B \to VP$ (V denotes a vector meson), such as $B \to \rho\pi$, $\omega\pi$, ρK, et cetera, which are well understood within the standard model (SM). However, among the $B \to PP$ decay modes, the BR of the decay modes $B^{\pm(0)} \to \eta' K^{\pm(0)}$ is found to be still larger than that expected within the SM. For the last several years the experimental results of unexpectedly large branching ratios (BRs) for $B \to \eta' K$ decays have drawn a lot of theoretical attention. The observed BRs for $B^{\pm} \to \eta' K^{\pm}$ in three different experiments are [1–3]

$$
\begin{aligned}
\mathcal{B}(B^{\pm} \to \eta' K^{\pm}) &= (77.9^{+6.2+9.3}_{-5.9-8.7}) \times 10^{-6} \quad \text{[BELLE]}, \\
&= (76.9 \pm 3.5 \pm 4.4) \times 10^{-6} \quad \text{[BABAR]}, \\
&= (80^{+10}_{-9} \pm 7) \times 10^{-6} \quad \text{[CLEO]}. \qquad (1)
\end{aligned}
$$

Many theoretical efforts have been made to explain the large BRs: for instance, approaches using the anomalous g–g–η' coupling [4–7], high charm content in η' [8–10], the spectator hard scattering mechanism [11,12], flavor U(3) symmetry [13], the QCD factorization (QCDF) approach [14], the perturbative QCD (PQCD) approach [15] and approaches to invoke new physics [16–20].

In earlier works on non-leptonic decays of B mesons, the factorization approximation, based on the color transparency argument, was usually assumed to estimate the hadronic matrix elements which are inevitably involved in theoretical calculations of the decay amplitudes for these processes. This naive factorization approach ignores the non-factorizable contributions from the soft interactions in the initial and final states. In order to compensate the non-factorizable contributions, the naive factorization scheme has been generalized by introducing the effective number of colors N_c as a phenomenological parameter. In this generalized factorization, the renormalization scheme and scale dependence in the hadronic matrix elements has been resolved [21].

Theoretically, the QCDF approach has provided a novel method to study non-leptonic B decays. In this approach, the naive factorization contributions become the leading term and as sub-leading contributions, radiative

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corrections from hard gluon exchange can be systematically calculated by using the perturbative QCD method in the heavy quark limit, where suppressed power corrections of $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ are neglected. Since the non-factorizable contributions in the naive factorization, such as the contributions from hard scattering with the spectator quark in the B meson and the contributions from weak annihilation, can be perturbatively computed, the phenomenological parameter N_c used in the generalized factorization scheme is no longer needed to compensate the non-factorizable contributions.

However, in reality the b quark is not very heavy so that the power corrections in $1/m_b$, particularly the chirally enhanced corrections, would not be negligible. The chirally enhanced corrections come from twist-3 light cone distribution amplitudes (LCDAs), but unfortunately the QCDF breaks down at twist-3 level because a logarithmic divergence appears in the hard spectator scattering at the end-point of the twist-3 LCDAs. A similar divergence also appears in the weak annihilation contributions. It is customary to phenomenologically treat these two end-point divergences by introducing model-dependent parameters [22]: $X_{\rm H}$ for the hard spectator scattering contributions and $X_{\rm A}$ for the weak annihilation contributions. Thus, it would be a less reliable case if these non-perturbative contributions of $X_{\rm H}$ and $X_{\rm A}$ become too large compared with the leading power radiative corrections. Since the prediction of the BRs for $B \to PP$ and $B \to VP$ decays strongly depend on the parameters $X_{\rm H}$ and $X_{\rm A}$, it is essential to reliably estimate the effects of $X_{\rm H}$ and $X_{\rm A}$.

In this work we study the decay processes $B^{\pm(0)} \to \eta' K^{\pm(0)}$ in the QCDF approach. In order to determine the parameters $X_{\rm H}$ and $X_{\rm A}$ more reliably, we use the global analysis as used in [23]. However, our global analysis differs from that used in [23], in the sense that we exclude the decay modes whose (dominant) internal quark-level process is $b \to s\bar{s}s$: for example, $B \to \phi K$ and $B \to \eta^{(\prime)} M$, where M denotes a light meson, such as K, K^*. The reason for excluding such modes is that the recent Belle measurement of the large negative value of $\sin(2\phi_1)_{\phi K_s}$ (ϕ_1 is the angle of the unitarity triangle) through the time-dependent decay process $B^0 \to \phi K_s$ shows a possibility that there may be new physics effects on the quark-level process $b \to s\bar{s}s$ [24]. Thus, to be conservative, in our global analysis within the SM, all the decay channels whose (dominant) quark-level process is $b \to s\bar{s}s$ are excluded so that parameters $X_{\rm H}$ and $X_{\rm A}$ can be determined without new physics prejudice when using the global fit. For the analysis, we will use twelve $B \to PP$ and VP decay modes, including $B \to \pi\pi$, πK, $\rho\pi$, ρK, $\omega\pi$, ωK. It turns out that both cases of the large and small $X_{\rm H,A}$ effects are allowed by the global fit. We will take into account both possibilities. In particular, motivated by the recent Belle result on $\sin(2\phi_1)_{\phi K_s}$, we will seriously examine new physics effects on the large BRs for $B \to \eta' K$. As specific examples of new physics models, we will present both R-parity violating (RPV) supersymmetry (SUSY) and R-parity conserving (RPC) SUSY scenario.

We would like to comment on a possible flavor-singlet contribution to $B \to \eta^{(\prime)} K$ decays proposed in [14]. Even though in principle the flavor-singlet contribution might give a sizable effect on the BRs of $B \to \eta^{(\prime)} K$, the calculation including only the other contributions, such as weak annihilation, could already reproduce the experimental results within large uncertainties as shown in [14]. These large uncertainties arise mainly from weak annihilation and the error on the strange quark mass, but the completely unknown flavor-singlet contribution is also a primary source of large uncertainties. Thus, we will not take into account this unknown flavor-singlet contribution in our analysis. After determining this flavor-singlet effect by certain methods (for instance, see [25]), one can definitely further improve the theoretical estimations.

This work is organized as follows. In Sect. 2, we introduce the framework: the effective Hamiltonian for non-leptonic charmless B decays and the QCDF approach. The decay amplitudes for $B \to \eta^{(\prime)} K$ in the QCDF are presented in Sect. 3. In Sect. 4, we discuss the global analysis for $B \to PP$ and VP decays and calculate the BRs for $B \to \eta' K$ decays as well as $B \to \phi K$ by using the inputs determined from the global analysis. We present the results for both cases of the large and small $X_{\rm H,A}$ effects. In particular, in the case of the small $X_{\rm H,A}$ effects, two new physics scenarios (RPV SUSY and RPC SUSY) are considered. We conclude the analysis in Sect. 5.

2 Framework

The effective Hamiltonian for hadronic charmless B decays can be written as

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \Bigg\{ \sum_{p=u,c} V_{pb} V_{pq}^* \Big[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{k=3}^{10} C_k(\mu) O_k(\mu) \Big] - V_{tb} V_{tq}^* \Big[C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \Big] \Bigg\} + {\rm H.c.} \quad (q = d, s), \tag{2}$$

where the dimension-6 local operators O_i are given by

$$\begin{aligned}
O_1^u &= (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A}, \\
O_1^c &= (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta c_\beta)_{V-A}, \\
O_2^u &= (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A}, \\
O_2^c &= (\bar{c}_\alpha b_\beta)_{V-A} (\bar{q}_\beta c_\alpha)_{V-A}, \\
O_3 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A}, \\
O_4 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A}, \\
O_5 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A}, \\
O_6 &= (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A},
\end{aligned}$$

$$O_7 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A},$$
$$O_8 = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A},$$
$$O_9 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A},$$
$$O_{10} = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A},$$
$$O_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu}(1+\gamma_5) b_\alpha F_{\mu\nu},$$
$$O_{8g} = \frac{g}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu}(1+\gamma_5) t^a_{\alpha\beta} b_\beta G^a_{\mu\nu}, \quad (3)$$

where q' denotes all the active quarks at the scale $\mu = \mathcal{O}(m_b)$, i.e., $q' = u, d, s, c, b$. The operators O_1^p, O_2^p are the tree operators, O_{3-6} are the strong penguin operators, O_{7-10} are the electroweak penguin operators, and $O_{7\gamma}$, O_{8g} are the magnetic penguin operators. The Wilson coefficients (WCs) $C_i(\mu)$ are obtained by running the renormalization group equations from the weak scale down to the scale μ. We will use the WCs evaluated to the next-to-leading logarithmic order in the NDR scheme, as given in [26].

In the QCDF approach, in the heavy quark limit $m_b >> \Lambda_{\rm QCD}$, the hadronic matrix element for $B \to M_1 M_2$ due to a particular operator O_i can be written in the form

$$\langle M_1 M_2 | O_i | B \rangle = \langle M_1 M_2 | O_i | B \rangle_{\rm NF} \cdot \left[1 + \sum_n r_n (\alpha_{\rm s})^n + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b} \right) \right], \quad (4)$$

where $\langle M_1 M_2 | O_i | B \rangle_{\rm NF}$ denotes the naive factorization result. The second and third term in the square bracket represent the radiative corrections in $\alpha_{\rm s}$ and the power corrections in $\Lambda_{\rm QCD}/m_b$. The decay amplitudes for $B \to M_1 M_2$ can be expressed as

$$\mathcal{A}(B \to M_1 M_2) = \mathcal{A}^f(B \to M_1 M_2) + \mathcal{A}^a(B \to M_1 M_2) , \quad (5)$$

where

$$\mathcal{A}^f(B \to M_1 M_2) = \frac{G_{\rm F}}{\sqrt{2}} \sum_{p=u,c} \sum_{i=1}^{10} V_{pb} V^*_{pq}\; a_i^p \langle M_1 M_2 | O_i | B \rangle_{\rm NF} ,$$
$$\mathcal{A}^a(B \to M_1 M_2) = \frac{G_{\rm F}}{\sqrt{2}}\; f_B f_{M_1} f_{M_2} \sum_{p=u,c} \sum_{i=1}^{10} V_{pb} V^*_{pq}\; b_i . \quad (6)$$

Here $\mathcal{A}^f(B \to M_1 M_2)$ includes vertex corrections, penguin corrections, and hard spectator scattering contributions which are absorbed into the QCD coefficients a_i, and $\mathcal{A}^a(B \to M_1 M_2)$ includes weak annihilation contributions which are absorbed into the parameter b_i. For the explicit expressions of a_i and b_i, we refer to [22,26].

It is well known [22] that both in the hard spectator scattering and in the annihilation contributions there appears logarithmic divergence in the end-point region. In [22], Beneke et al. introduced phenomenological parameters for the end-point divergent integrals:

$$X_{\rm H,A} \equiv \int_0^1 \frac{{\rm d}x}{x} \equiv \left(1 + \rho_{\rm H,A} {\rm e}^{{\rm i}\phi_{\rm H,A}}\right) \ln \frac{m_B}{\Lambda_h} , \quad (7)$$

where $X_{\rm H}$ and $X_{\rm A}$ denote the hard spectator scattering contribution and the annihilation contribution, respectively. Here the phases $\phi_{\rm H,A}$ are arbitrary, $0^\circ \le \phi_{\rm H,A} \le 360^\circ$, and the parameter $\rho_{\rm H,A} \le 1$ and the scale $\Lambda_h = 0.5\,{\rm GeV}$ assumed phenomenologically [22]. In principle, the parameters $\rho_{\rm H,A}$ and $\phi_{\rm H,A}$ for $B \to PP$ decays can be different from those for $B \to VP$ decays. Thus, for $B \to PP$ and VP decays, from the end-point divergent integrals, eight new parameters are introduced: $\rho^{PP}_{\rm H,A}$, $\phi^{PP}_{\rm H,A}$ for $B \to PP$, and $\rho^{VP}_{\rm H,A}$, $\phi^{VP}_{\rm H,A}$ for $B \to VP$.

3 Decay processes $B^{\pm(0)} \to \eta^{(\prime)} K^{\pm(0)}$ in the QCDF approach

The decay amplitudes for $B^- \to \eta^{(\prime)} K^-$ and $\bar{B}^0 \to \eta^{(\prime)} \bar{K}^0$ in the QCDF are given by

$$\begin{aligned}
&\mathcal{A}(B^- \to \eta^{(\prime)} K^-) \\
&= -{\rm i}\frac{G_{\rm F}}{\sqrt{2}} f_K F_0^{B\to\eta^{(\prime)}}(m_K^2)(m_B^2 - m^2_{\eta^{(\prime)}}) \\
&\times [V_{ub}V^*_{us}(a_1' + a_4^{u\,\prime} + a_{10}^{u\,\prime} + (a_6^{u\,\prime} + a_8^{u\,\prime})R_1) \\
&+ V_{cb}V^*_{cs}(a_4^{c\,\prime} + a_{10}^{c\,\prime} + (a_6^{c\,\prime} + a_8^{c\,\prime})R_1)] \\
&- {\rm i}\frac{G_{\rm F}}{\sqrt{2}} F_0^{B\to K}(m^2_{\eta^{(\prime)}})(m_B^2 - m_K^2) \\
&\times \left\{ f^u_{\eta^{(\prime)}} \left[V_{ub}V^*_{us}\left(a_2 + 2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) \right.\right.\right. \\
&\qquad \left. - \left(a_6^u - \frac{1}{2}a_8^u \right) R_3 \right) \\
&+ V_{cb}V^*_{cs}\left(2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) - \left(a_6^c - \frac{1}{2}a_8^c \right) R_3 \right) \Big] \\
&+ f^s_{\eta^{(\prime)}} \left[V_{ub}V^*_{us}\left(a_3 + a_4^u - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) \right.\right. \\
&\qquad \left. + \left(a_6^u - \frac{1}{2}a_8^u \right) R_3 \right) \\
&+ V_{cb}V^*_{cs}\left(a_3 + a_4^c - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) \right. \\
&\qquad \left.\left.\left. + \left(a_6^c - \frac{1}{2}a_8^c \right) R_3 \right) \right] \right\} \\
&- {\rm i}\frac{G_{\rm F}}{\sqrt{2}} f_B f_K \left(f^u_{\eta^{(\prime)}} + f^s_{\eta'} \right) \\
&\times [V_{ub}V^*_{us} b_2 + (V_{ub}V^*_{us} + V_{cb}V^*_{cs})(b_3 + b_3^{\rm ew})] ,
\end{aligned} \quad (8)$$

$$
\begin{aligned}
&\mathcal{A}(B^- \to \eta^{(\prime)} K^-) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\eta^{(\prime)}}(m_K^2)(m_B^2 - m_{\eta^{(\prime)}}^2) \\
&\times[V_{ub}V_{us}^*(a_1' + a_4^{u\,\prime} + a_{10}^{u\,\prime} + (a_6^{u\,\prime} + a_8^{u\,\prime})R_1) \\
&+V_{cb}V_{cs}^*(a_4^{c\,\prime} + a_{10}^{c\,\prime} + (a_6^{c\,\prime} + a_8^{c\,\prime})R_1)] \\
&-i\frac{G_F}{\sqrt{2}} F_0^{B\to K}(m_{\eta^{(\prime)}}^2)(m_B^2 - m_K^2) \\
&\times\Big\{ f_{\eta^{(\prime)}}^u \,[V_{ub}V_{us}^*\,(a_2 + 2a_3 - 2a_5 \\
&-\frac{1}{2}(a_7 - a_9) - \left(a_6^u - \frac{1}{2}a_8^u\right) R_3 \\
&+V_{cb}V_{cs}^*\left(2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) - \left(a_6^c - \frac{1}{2}a_8^c\right) R_3\right)\Big] \\
&+f_{\eta^{(\prime)}}^s \Big[V_{ub}V_{us}^*\Big(a_3 + a_4^u - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) \\
&+\left(a_6^u - \frac{1}{2}a_8^u\right) R_3\Big) \\
&+V_{cb}V_{cs}^*\Big(a_3 + a_4^c - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) \\
&+\left(a_6^c - \frac{1}{2}a_8^c\right) R_3\Big)\Big]\Big\} \\
&-i\frac{G_F}{\sqrt{2}} f_B f_K \left(f_{\eta^{(\prime)}}^u + f_{\eta'}^s\right) \\
&\times[V_{ub}V_{us}^* b_2 + (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\,(b_3 + b_3^{ew})]\ , \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{A}(\bar{B}^0 \to \eta^{(\prime)} \bar{K}^0) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B\to\eta^{(\prime)}}(m_K^2)(m_B^2 - m_{\eta^{(\prime)}}^2) \\
&\times[V_{ub}V_{us}^*(a_4^{u\,\prime} - \frac{1}{2}a_{10}^{u\,\prime} + (a_6^{u\,\prime} - \frac{1}{2}a_8^{u\,\prime})R_2) \\
&+V_{cb}V_{cs}^*(a_4^{c\,\prime} - \frac{1}{2}a_{10}^{c\,\prime} + (a_6^{c\,\prime} - \frac{1}{2}a_8^{c\,\prime})R_2)] \\
&-i\frac{G_F}{\sqrt{2}} F_0^{B\to K}(m_{\eta'}^2)(m_B^2 - m_K^2) \\
&\times\Big\{ f_{\eta^{(\prime)}}^u \Big[V_{ub}V_{us}^*\Big(a_2 + 2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) \\
&-\left(a_6^u - \frac{1}{2}a_8^u\right) R_3\Big) \\
&+V_{cb}V_{cs}^*\left(2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) - \left(a_6^c - \frac{1}{2}a_8^c\right) R_3\right)\Big] \\
&+f_{\eta^{(\prime)}}^s \Big[V_{ub}V_{us}^*\Big(a_3 + a_4^u - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) \\
&+\left(a_6^u - \frac{1}{2}a_8^u\right) R_3\Big) \\
&+V_{cb}V_{cs}^*\Big(a_3 + a_4^c - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) \\
&+\left(a_6^c - \frac{1}{2}a_8^c\right) R_3\Big)\Big]\Big\} \\
&-i\frac{G_F}{\sqrt{2}} f_B f_K \left(f_{\eta^{(\prime)}}^u + f_{\eta^{(\prime)}}^s\right) \\
&\times(V_{ub}V_{us}^* + V_{cb}V_{cs}^*)\left(b_3 - \frac{1}{2}b_3^{ew}\right)\ , \qquad (10)
\end{aligned}
$$

where

$$
R_{1(2)} = \frac{2m_{K^{(0)}}^2}{(m_b - m_{u(d)})(m_{u(d)} + m_s)}\ , \qquad
R_3 = \frac{2m_{\eta'}^2}{2m_s(m_b - m_s)}\ . \qquad (11)
$$

The coefficients $a_i^{(\prime)}$ and b_i are expressed as

$$
\begin{aligned}
a_1^{(\prime)} &= C_1 + \frac{C_2}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_M + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right], \\
a_2 &= C_2 + \frac{C_1}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_M + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right], \\
a_3 &= C_3 + \frac{C_4}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_M + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right], \\
a_4^{p\ (\prime)} &= C_4 + \frac{C_3}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_M + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right] \\
&\quad + \frac{C_F\alpha_s}{4\pi N_c}P_{M,2}^p, \\
a_5 &= C_5 \\
&\quad + \frac{C_6}{N_c}\left[1 - \frac{C_F\alpha_s}{4\pi}\left(V_M + 12 + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right], \\
a_6^{p\ (\prime)} &= C_6 + \frac{C_5}{N_c}\left[1 - 6\frac{C_F\alpha_s}{4\pi}\right] + \frac{C_F\alpha_s}{4\pi N_c}P_{M,3}^p, \\
a_7 &= C_7 \\
&\quad + \frac{C_8}{N_c}\left[1 - \frac{C_F\alpha_s}{4\pi}\left(V_M + 12 + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right], \\
a_8^{p\ (\prime)} &= C_8 + \frac{C_7}{N_c}\left[1 - 6\frac{C_F\alpha_s}{4\pi}\right] + \frac{\alpha}{9\pi N_c}P_{M,3}^{p,\mathrm{ew}}, \\
a_9 &= C_9 + \frac{C_{10}}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_M + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right], \\
a_{10}^{p\ (\prime)} &= C_{10} + \frac{C_9}{N_c}\left[1 + \frac{C_F\alpha_s}{4\pi}\left(V_M + \frac{4\pi^2}{N_c}H(BM_1,M_2)\right)\right] \\
&\quad + \frac{\alpha}{9\pi N_c}P_{M,2}^{p,\mathrm{ew}}, \\
b_2 &= \frac{C_F}{N_c^2}C_2A^i, \\
b_3 &= \frac{C_F}{N_c^2}[C_3A^i + A^f(C_5 + N_cC_6)], \\
b_3^{\mathrm{ew}} &= \frac{C_F}{N_c^2}[C_9A^i + A^f(C_7 + N_cC_8)], \qquad (12)
\end{aligned}
$$

where the superscript p is u or c, and the color factor $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$. The vertex parameter V_M and the hard spectator scattering parameter $H(BM_1, M_2)$, and the weak annihilation parameters A^i, A^f are given by [22, 26]

$$
\begin{aligned}
V_M &= 12\ln\frac{m_b}{\mu} - 18 + \int_0^1 \mathrm{d}x\ g(x)\Phi_M(x), \\
&H(BM_1, M_2) \\
&= \frac{f_B f_{M_1}}{m_B^2 F_0^{B\to M_1}}\int_0^1 \mathrm{d}\xi\frac{\Phi_B(\xi)}{\xi}\int_0^1 \mathrm{d}x\frac{\Phi_{M_2}(x)}{(1-x)}
\end{aligned}
$$

$$\times \int_0^1 \mathrm{d}y \left[\frac{\Phi_{M_1}(y)}{(1-y)} + \frac{2\mu_{M_1}}{m_b}\frac{(1-x)}{x}\frac{\Phi^p_{M_1}(y)}{(1-y)}\right],$$
$$A^i \approx \pi\alpha_\mathrm{s}\left[18\left(X_\mathrm{A} - 4 + \frac{\pi^2}{3}\right) + 2r_\chi^2 X_\mathrm{A}^2\right],$$
$$A^f \approx 12\pi\alpha_\mathrm{s} r_\chi X_\mathrm{A}(2X_\mathrm{A} - 1), \quad (13)$$

where $g(x) = 3\left(\frac{1-2x}{1-x}\right)\ln x - 3\mathrm{i}\pi$ and $\mu_P = \frac{m_P^2}{m_1+m_2}$ (m_1 and m_2 are current quark masses of the valence quarks of the meson P) and the chirally enhanced factor $r_\chi = \frac{2\mu_P}{m_b}$. For the chirally enhanced parameter r_χ, we will take $r_\chi^{\eta'}\left(1 - \frac{f^u_{\eta'}}{f^s_{\eta'}}\right) = r_\chi^\pi = r_\chi^K \equiv r_\chi$ as in [26]. $X_\mathrm{A} \equiv \int_0^1 \frac{\mathrm{d}x}{x}$ is a logarithmically divergent integral. For the wave function $\Phi_B(\xi)$ of the B meson, we take the following parametrization:

$$\int_0^1 \mathrm{d}\xi\, \frac{\Phi_B(\xi)}{\xi} \equiv \frac{m_B}{\lambda_B}, \quad (14)$$

where the parameter λ_B is estimated as $\lambda_B = (350 \pm 150)$ MeV [22]. For the K and η' meson, we use the asymptotic forms of the LCDAs [22]:

$$\Phi_K(x) = \Phi_{\eta'} = 6x(1-x),$$
$$\Phi^p_K(x) = \Phi^p_{\eta'}(x) = 1, \quad (15)$$

where $\Phi_M(x)$ and $\Phi^p_M(x)$ are the leading twist LCDAs and twist-3 LCDAs of the meson $M = K,\ \eta'$, respectively. The explicit expressions of the QCD penguin parameters $P^p_{M,i}$ and the electroweak penguin parameters $P^{p,\mathrm{ew}}_{M,i}$ can be found in [22,26]. The coefficients a_i and a'_i in (9) and (10) include the different vertex and hard spectator scattering contributions: for a_i, $V_M = V_{\eta'}$ and $H(BM_1, M_2) = H(BK, \eta')$, while for a'_i, $V_M = V_K$ and $H(BM_1, M_2) = H(B\eta', K)$.

Note that in (13) the hard spectator scattering parameter $H(BM_1, M_2)$ includes a logarithmically divergent integral $\int_0^1 \mathrm{d}y/(1-y)$ which arises from the twist-3 contribution, and the weak annihilation parameters A^i and A^f include another logarithmically divergent integral X_A.

For the η–η' mixing, we use the following relation:

$$|\eta\rangle = \cos\theta_8|\eta_8\rangle - \sin\theta_0|\eta_0\rangle,$$
$$|\eta'\rangle = \sin\theta_8|\eta_8\rangle + \cos\theta_0|\eta_0\rangle, \quad (16)$$

where η_8 and η_0 are the flavor SU(3) octet and single, respectively. The mixing angles are $\theta_8 \approx -22.2°$ and $\theta_0 \approx -9.1°$ [27]. The decay constants and form factors relevant for the $B \to \eta^{(\prime)}$ transitions are given by

$$f^u_\eta = \frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0,$$
$$f^s_\eta = -2\frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0,$$
$$f^u_{\eta'} = \frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0,$$
$$f^s_{\eta'} = -2\frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0,$$
$$F^{B\eta}_{0,1} = F^{B\pi}_{0,1}\left(\frac{\cos\theta_8}{\sqrt{6}} - \frac{\sin\theta_8}{\sqrt{3}}\right),$$
$$F^{B\eta'}_{0,1} = F^{B\pi}_{0,1}\left(\frac{\sin\theta_8}{\sqrt{6}} + \frac{\cos\theta_8}{\sqrt{3}}\right). \quad (17)$$

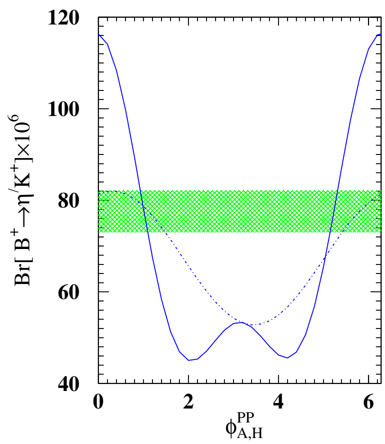

Fig. 1. Dependence of the BR for $B^+ \to \eta' K^+$ on ϕ^{PP}_A (solid line) or ϕ^{PP}_H (dotted line). Here the following values of the other parameters are used: $\rho^{PP}_\mathrm{A} = \rho^{PP}_\mathrm{H} = 1$ (for both lines), $\phi^{PP}_\mathrm{H} = -23°$ (for the solid line), $\phi^{PP}_\mathrm{A} = 57°$ (for the dotted line). The shaded region is allowed by the experimental data

4 Global analysis and numerical result

In order to calculate the BRs for B decays in the QCDF approach, various input parameters are needed, such as the CKM matrix elements, decay constants, transition form factors, LCDAs, and so on. Among those input parameters, it is urgently essential to reliably estimate the annihilation parameter X_A and the hard spectator scattering parameter X_H: more specifically, ρ_A, ϕ_A, ρ_H, and ϕ_H, because the predicted BRs strongly depend on the parameters X_A and X_H (see Figs. 1 and 2). Unfortunately, within the QCDF scheme, X_A and X_H are purely phenomenological parameters, so there is no definite way to determine them. Therefore, in order to determine the values of ρ_A, ϕ_A, ρ_H, and ϕ_H more reliably, in this work we follow the global analysis, used in [23]. For a detailed discussion of the method of the global fit we refer to [23]. As explained in Sect. 1, different from the global analysis used in [23], we do not include the decay modes, such as $B \to \phi K$ and $B \to \eta^{(\prime)} M$ (M denotes a light meson: e.g., K, K^*), whose (dominant) internal quark-level process is $b \to s\bar{s}s$. In this way, within the SM, the parameters ρ_A, ϕ_A, ρ_H, ϕ_H can be determined without new physics prejudice when using the global fit. Specifically, we use twelve decay modes, such as $B \to \pi\pi$, πK, $\rho\pi$, ρK, $\omega\pi$, and ωK, as listed in Table 1 [1–3].

In general, the new physics contributions to the $b \to s$ penguin processes can affect the $b \to s\bar{u}u$ and $b \to s\bar{d}d$ processes as well as the $b \to s\bar{s}s$ processes. In the R-parity

Table 1. Experimental data and the "good" fit values of CP-averaged branching ratios (in unit of 10^{-6}) for $B \to PP$ and VP decays used in our global analysis

Decay mode	Weighted average (Exp.)	Fit	Decay mode	Weighted average (Exp.)	Fit
$B^+ \to \pi^+\pi^0$	5.42 ± 0.83	4.85	$B^0 \to \pi^+\pi^-$	4.55 ± 0.44	4.75
$B^+ \to \pi^+ K^0$	20.8 ± 1.4	20.5	$B^0 \to \pi^+ K^-$	18.1 ± 0.8	18.9
$B^+ \to \pi^0 K^+$	12.7 ± 1.1	11.4	$B^0 \to \pi^0 K^0$	11.2 ± 1.4	8.5
$B^+ \to \pi^+\rho^0$	8.6 ± 2.0	7.6	$B^0 \to \pi^\pm\rho^\mp$	22.7 ± 2.5	23.5
$B^+ \to \omega\pi^+$	6.0 ± 0.9	6.45	$B^0 \to K^+\rho^-$	8.0 ± 1.7	9.5
$B^+ \to \omega K^+$	5.6 ± 0.9	5.25	$B^0 \to \omega K^0$	5.3 ± 1.5	4.45

violating SUSY case, as we shall see later, one can assume that the new physics effects appear only on the $b \to s\bar{s}s$ processes, because the new operators can be chosen to be relevant only to the $b \to s\bar{s}s$ processes. Thus, in the case of the R-parity violating SUSY scenario, our strategy of excluding the $b \to s\bar{s}s$ processes in the global analysis is justified. On the otherhand, in the R-parity conserving SUSY scenario, one may need more caution for selecting the relevant processes in the global fit. In order to make sure, one may need to consider both fits, *including* and *excluding* the $b \to s\bar{s}s$ processes. However, we shall see (Table 2) that in the case of large $X_{\rm A}$ and $X_{\rm H}$ effects, our fit (within the SM) is already in good agreement with the experimental data *including* the $b \to s\bar{s}s$ processes, such as the BRs of $B \to \eta' K$ and $B \to \phi K$ modes. Consequently in this case, there is no room for invoking new physics effects. However, a large negative value of $\sin(2\phi_1)_{\phi K_s}$ cannot be obtained. For the case with small $X_{\rm A}$ and $X_{\rm H}$ effects, it turns out to be almost unlikely to find any acceptable fit *including* the $b \to s\bar{s}s$ processes.

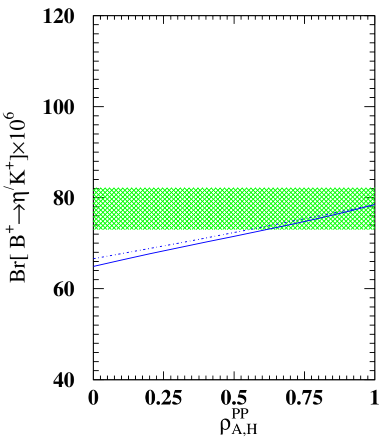

Fig. 2. Dependence of the BR for $B^+ \to \eta' K^+$ on $\rho_{\rm A}^{PP}$ (solid line) or $\rho_{\rm H}^{PP}$ (dotted line). Here the following values of the other parameters are used: $\phi_{\rm A}^{PP} = -23°$ and $\phi_{\rm H}^{PP} = 57°$ (for both lines), $\rho_{\rm H}^{PP} = 1$ (for the solid line), $\rho_{\rm A}^{PP} = 1$ (for the dotted line). The shaded region is allowed by the experimental data

First, we examine the dependence of the BR for $B^+ \to \eta' K^+$ on the effects of $X_{\rm A}$ and $X_{\rm H}$. Figure 1 shows $\mathcal{B}(B^+ \to \eta' K^+)$ versus $\phi_{\rm A}^{PP}$ (solid line) or $\phi_{\rm H}^{PP}$ (dotted line). In each case, $\phi_{\rm A}^{PP}$ or $\phi_{\rm H}^{PP}$ varies from 0 to 2π. For the solid line, other inputs are set as $\rho_{\rm A}^{PP} = \rho_{\rm H}^{PP} = 1$, $\phi_{\rm H}^{PP} = -23°$. For the dotted line, $\rho_{\rm A}^{PP} = \rho_{\rm H}^{PP} = 1$, $\phi_{\rm A}^{PP} = 57°$. We see that the predicted BR for $B^+ \to \eta' K^+$ strongly depends on $\phi_{\rm A}^{PP}$ and $\phi_{\rm H}^{PP}$. In particular, as the value of $\phi_{\rm A}^{PP}$ varies, the predicted BR can change by a factor of about 2.5 (e.g., from 45×10^{-6} to 116×10^{-6}). The allowed values of $\phi_{\rm A}^{PP}$ are in certain narrow regions which can be practically found by the global analysis. Similarly, Fig. 2 shows $\mathcal{B}(B^+ \to \eta' K^+)$ versus $\rho_{\rm A}^{PP}$ (solid line) or $\rho_{\rm H}^{PP}$ (dotted line). In each case, $\rho_{\rm A}^{PP}$ or $\rho_{\rm H}^{PP}$ varies from 0 to 1. The other inputs are taken $\phi_{\rm A}^{PP} = -23°$ and $\phi_{\rm H}^{PP} = 57°$ for both lines, $\rho_{\rm H}^{PP} = 1$ for the solid line, and $\rho_{\rm A}^{PP} = 1$ for the dotted line. The predicted BR for $B^+ \to \eta' K^+$ is also dependent on $\rho_{\rm A}^{PP}$ and $\rho_{\rm H}^{PP}$, but its dependence on $\rho_{\rm A,H}^{PP}$ is weaker than that on $\phi_{\rm A}^{PP}$. We notice that the prediction of $\mathcal{B}(B^+ \to \eta' K^+)$ is very sensitive to the effect of $X_{\rm A}$ through $\phi_{\rm A}^{PP}$. This feature also holds for the neutral mode $B^0 \to \eta' K^0$.

We find that the best fit [and also the "good" fit (see the discussions below Case 1)] of the global analysis favors large effects of the parameters $X_{\rm A}$ and $X_{\rm H}$. This tendency is consistent with the results of other previous works done in the QCDF scheme [23,28]. But, as mentioned in Sect. 1, if the non-perturbative effects of $X_{\rm A}$ and $X_{\rm H}$ are

Table 2. Experimental data and the prediction of the branching ratios (in unit of 10^{-6}) for $B \to \eta' K$ and $B \to \phi K$ decays. Here the inputs for the "good" fit are used. For comparison, the predicted BRs for three cases are also listed: (i) for $X_{\rm A} = X_{\rm H} = 0$, (ii) for only $X_{\rm A} = 0$, (iii) for only $X_{\rm H} = 0$

Decay mode	Exp. data	Prediction	$X_{\rm A} = X_{\rm H} = 0$	$X_{\rm A} = 0$ only	$X_{\rm H} = 0$ only
$B^+ \to \eta' K^+$	77.6 ± 4.6	78.5	31.2	53.4	53.5
$B^0 \to \eta' K^0$	65.0 ± 6.0	71.6	28.8	59.5	48.7
$B^+ \to \phi K^+$	9.3 ± 0.7	8.85	2.31	2.31	8.85
$B^0 \to \phi K^0$	8.2 ± 1.1	8.01	2.20	2.20	8.01

too large or dominant compared with the leading power radiative corrections, the theoretical predictions based on these effects would be less reliable and become questionable. Therefore, one can seriously ask the following question: Is it possible to find a global fit where the effects of $X_{\rm A}$ and $X_{\rm H}$ are rather small (so the theoretical predictions based on these effects would be more reliable), but its χ^2 value is still acceptably small? In fact, it turns out that such an acceptable fit with the small effects of $X_{\rm A}$ and $X_{\rm H}$ can be found.

For calculation of the BRs for $B^{+(0)} \to \eta' K^{+(0)}$, we take into account two different possibilities as discussed above: Case 1 with the large $X_{\rm A}$ and $X_{\rm H}$ effects (favored by the best and "good" fit, but less reliable), Case 2 with the small $X_{\rm A}$ and $X_{\rm H}$ effects (more reliable).

The theoretical input parameters used in our global analysis are scanned in the following ranges [22]: first, the CKM parameters (λ, A, ϕ_3, $|V_{ub}|$) are set to be completely free because they are fundamental theory parameters. (Here $\lambda \equiv |V_{us}|$ and the parameter A is defined by $A\lambda^2 = |V_{cb}|$ and ϕ_3 is the angle of the unitarity triangle.) For the other parameters,

$$\begin{aligned} \mu &= \left(\frac{m_b}{2},\ 2m_b\right) = (2.1\text{–}8.4)\,\text{GeV}, \\ m_s &= (110 \pm 25)\,\text{MeV}, \\ f_B &= (180 \pm 40)\,\text{MeV}, \quad \lambda_B = (350 \pm 150)\,\text{MeV}, \\ F^{B\pi} &= 0.28 \pm 0.05, \quad R_{\pi K} = 0.9 \pm 0.1, \\ A^{B\rho} &= 0.37 \pm 0.06, \end{aligned} \tag{18}$$

where $R_{\pi K} \equiv (f_\pi F^{BK})/(f_K F^{B\pi})$. f_B is the B meson decay constant, and $F^{B\pi}$ and $A^{B\rho}$ are the form factors for the transition $B \to \pi$ and $B \to \rho$, respectively. The scanning ranges of the parameters $\rho_{\rm H,A}$ and $\phi_{\rm H,A}$ are given below for each of Case I and II.

Case 1 with the large $X_{\rm A}$ and $X_{\rm H}$ effects

We first try to find the best fit of the global analysis for the twelve $B \to PP$ and VP decay channels shown in Table 1. Our result shows that based on the theoretical inputs for the best fit (with $\chi^2_{\rm min} = 7.5$), the predicted BR for $B^+ \to \eta' K^+$ is consistent with the experimental data, but the prediction of $\mathcal{B}(B^+ \to \phi K^+)$ is too small compared with the data (for the data, see Table 2), as the best fit predicted BRs are

$$\begin{aligned} \mathcal{B}(B^+ \to \eta' K^+) &= 74.7 \times 10^{-6}, \\ \mathcal{B}(B^+ \to \phi K^+) &= 4.02 \times 10^{-6}. \end{aligned} \tag{19}$$

It happens because the internal interference between different contributions (e.g., contributions from the hard spectator scattering and weak annihilation) to the decay amplitude for $B^+ \to \eta' K^+$ is quite different from that for $B^+ \to \phi K^+$ (for example, see Table 2). It turns out that it is possible to obtain successful fits to all $B \to \eta' K$ and $B \to \phi K$ data, if one assumes that there are new physics effects on the quark-level process $b \to s\bar{s}s$. We will discuss this possibility later in SUSY scenarios.

Since the input parameter values for the best fit are not consistent with the experimental result such as the BR for $B \to \phi K$, we investigate another possibility that there may exist a "good" fit for which the predictions based on the inputs are consistent with the experimental measurements including $B \to \eta' K$ and $B \to \phi K$, and whose $\chi^2_{\rm min}$ value is still quite small. In fact, we find such a "good" fit with $\chi^2_{\rm min} = 8.6$ for the twelve decay modes. Notice that this $\chi^2_{\rm min}$ value is not much different from that of the best fit. In Table 1, we list the "good" fit values of the BRs for the relevant $B \to PP$ and VP decay modes. The corresponding theoretical inputs are given by

$$\begin{aligned} &\lambda = 0.2205, \quad A = 0.814, \quad \phi_3 = 72^\circ, \\ &|V_{ub}| = 3.49 \times 10^{-3}, \\ &\mu = 2.1\,\text{GeV}, \quad m_s(2\,\text{GeV}) = 85\,\text{MeV}, f_B = 220\,\text{MeV}, \\ &\lambda_B = 200\,\text{MeV}, \\ &F^{B\pi} = 0.23, \quad R_{\pi K} = 1, \quad A^{B\rho} = 0.31, \\ &\rho_{\rm A}^{PP} = \rho_{\rm A}^{VP} = \rho_{\rm H}^{PP} = \rho_{\rm H}^{VP} = 1, \\ &\phi_{\rm A}^{PP} = 57^\circ, \quad \phi_{\rm A}^{VP} = 52^\circ, \quad \phi_{\rm H}^{PP} = -23^\circ, \quad \phi_{\rm H}^{VP} = 180^\circ, \end{aligned} \tag{20}$$

where the scanning ranges of the parameters $\rho_{\rm A,H}^{PP,VP}$ and $\phi_{\rm A,H}^{PP,VP}$ are $0 \le \rho_{\rm A,H}^{PP,VP} \le 1$ and $\phi_{\rm A,H}^{PP,VP}$ set to be free.

Indeed the BRs for $B^{+(0)} \to \eta' K^{+(0)}$ and $B^{+(0)} \to \phi K^{+(0)}$ calculated by using the above inputs are in good agreement with the experimental measurements as shown in Table 2. Therefore, our result shows that the large BRs for the processes $B^{+(0)} \to \eta' K^{+(0)}$ as well as the BRs for $B^{+(0)} \to \phi K^{+(0)}$ can be consistently understood, based on the global analysis for $B \to PP$ and VP decays, where the values of the pure phenomenological parameters $\rho_{\rm A,H}^{PP}$, $\rho_{\rm A,H}^{VP}$, $\phi_{\rm A,H}^{PP}$ and $\phi_{\rm A,H}^{VP}$ are reasonably determined. The BRs for $B^{+(0)} \to \eta K^{+(0)}$ are estimated as $(1 \sim 2) \times 10^{-6}$ which are also consistent with the data [1–3]: $\mathcal{B}(B^+ \to \eta K^+) = (3.7 \pm 0.7) \times 10^{-6}$ and $\mathcal{B}(B^0 \to \eta K^0) = (2.9 \pm 1.0) \times 10^{-6}$.

However, we note that the inputs given in (20) provide large effects of $X_{\rm A}$ and $X_{\rm H}$: e.g., $\rho_{\rm A,H}^{PP} = \rho_{\rm A,H}^{VP} = 1$ [see (7)]. In order to explicitly estimate the effects of $X_{\rm A}$ and $X_{\rm H}$, we also examine three interesting cases. In the fourth column of Table 2, the BRs for $B \to \eta' K$ and $B \to \phi K$ are calculated for $X_{\rm A} = X_{\rm H} = 0$. Similarly, in the fifth and last column, those BRs are calculated under the assumption of $X_{\rm A} = 0$ or $X_{\rm H} = 0$, respectively. Note that for $B^{+(0)} \to \phi K^{+(0)}$ modes, all the numbers of Table 2 are obtained for $X_{\rm H} = 0$, because of the input value $\phi_{\rm H}^{VP} = 180^\circ$ for the "good" fit as in (20). Thus, just for illustration, we re-examine the above three cases for $B^{+(0)} \to \phi K^{+(0)}$ modes by setting $\phi_{\rm H}^{VP} = 150^\circ$ (i.e., $X_{\rm H} \neq 0$ now). Then we find that the BRs of $B^{+(0)} \to \phi K^{+(0)}$ are

(i) 7.81 (7.05) for $X_{\rm A} \neq X_{\rm H} \neq 0$,
(ii) 2.31 (2.20) for $X_{\rm A} = X_{\rm H} = 0$,
(iii) 1.96 (1.87) for $X_{\rm A} = 0$ only,
(iv) 8.85 (8.01) for $X_{\rm H} = 0$ only.

We see that the contributions from the terms involving $X_{\rm A}$ and $X_{\rm H}$ are quite large for both $B \to \eta' K$ and

$B \to \phi K$ decays. In particular, for $\mathcal{B}(B^{+(0)} \to \phi K^{+(0)})$ the contribution of $X_{\rm A}$ (i.e., weak annihilation contribution) dominates over all the other contributions. It is clear that the internal interference between the effects of $X_{\rm A}$ and $X_{\rm H}$ on $B \to \eta' K$ is constructive, while (if there is any) that on $B \to \phi K$ is destructive. It should be stressed that in this scenario (i.e., with the large effect of $X_{\rm A,H}$ allowed by the "good" fit), there is no room for invoking new physics effects on the quark-level process $b \to s\bar{s}s$, which is implied by the *large negative* value of $\sin(2\phi_1)$ recently measured by Belle [24].

Case 2 with the small $X_{\rm A}$ and $X_{\rm H}$ effects

As already emphasized, if the non-perturbative contributions of $X_{\rm A}$ and $X_{\rm H}$ are too large, the predictions based on these contributions become less reliable and suspicious. However, in Case 1, we noticed that the contribution of $X_{\rm A}$ is very large, especially for $B^{+(0)} \to \phi K^{+(0)}$ modes. Therefore, it is natural to investigate presumably more reliable scenarios, where the effects of $X_{\rm A}$ and $X_{\rm H}$ are rather small or at least not dominant.

Using the global analysis for the twelve decay modes shown in Table 1, we find such a fit (with $\chi^2_{\rm min} = 18.3$) with the (relatively) small $X_{\rm A}$ and $X_{\rm H}$ effects. The corresponding theoretical inputs for this fit are as follows:

$$\begin{aligned} \lambda &= 0.2198, \quad A = 0.868, \quad \phi_3 = 86.8^\circ, \\ |V_{ub}| &= 3.35 \times 10^{-3}, \\ \mu &= 2.1\,{\rm GeV}, \quad m_s(2\,{\rm GeV}) = 85\,{\rm MeV}, \\ f_B &= 220\,{\rm MeV}, \\ F^{B\pi} &= 0.249, \quad R_{\pi K} = 1, \quad A^{B\rho} = 0.31, \\ \rho_{\rm A}^{PP} &= 0, \quad \rho_{\rm A}^{VP} = 0.5, \quad \rho_{\rm H}^{PP} = 1, \quad \rho_{\rm H}^{VP} = 0.746, \\ \phi_{\rm A}^{VP} &= -6^\circ, \quad \phi_{\rm H}^{VP} = \phi_{\rm H}^{PP} = 180^\circ, \end{aligned} \tag{21}$$

where the scanning ranges of the parameters $\rho_{\rm H}^{PP,VP}$ and $\phi_{\rm A}^{PP,VP}$ are: $0 \le \rho_{\rm H}^{PP,VP} \le 1$ and $\phi_{\rm A}^{PP,VP}$ set to be free. $\phi_{\rm H}^{PP,VP}$ and $\rho_{\rm A}^{PP,VP}$ are chosen to be $\phi_{\rm H}^{PP,VP} = 180^\circ$, $\rho_{\rm A}^{PP} = 0$ and $\rho_{\rm A}^{VP} = 0.5$ in order to find out an acceptable fit with as small $X_{\rm A}$, $X_{\rm H}$ effects as possible. Note that in this case the effect of the weak annihilation parameter $X_{\rm A}$ is relatively small (i.e., $\rho_{\rm A}^{PP} = 0$ and $\rho_{\rm A}^{VP} = 0.5$), and the effect of the hard spectator scattering parameter $X_{\rm H}$ is very small, because $\rho_{\rm H}^{PP} = 1$, $\rho_{\rm H}^{VP} = 0.746$, and $\phi_{\rm H}^{PP} = \phi_{\rm H}^{VP} = 180^\circ$ so that the terms 1 and $\rho_{\rm H} {\rm e}^{{\rm i}\phi_{\rm H}}$ in $X_{\rm H}$ [see (7)] cancel each other.

Based on the above inputs, the BRs for $B \to \eta' K$ and $B \to \phi K$ are predicted to be

$$\begin{aligned} \mathcal{B}(B^+ \to \eta' K^+) &= 51.1 \times 10^{-6}, \\ \mathcal{B}(B^0 \to \eta' K^0) &= 46.8 \times 10^{-6}, \\ \mathcal{B}(B^+ \to \phi K^+) &= 7.29 \times 10^{-6}, \\ \mathcal{B}(B^0 \to \phi K^0) &= 6.65 \times 10^{-6}. \end{aligned} \tag{22}$$

These BRs are quite small, especially for $B \to \eta' K$, compared with the experimental data, because of the small effects of $X_{\rm A}$ and $X_{\rm H}$ as well as the other fitted parameters such as ϕ_3. Since both processes $B \to \eta' K$ and $B \to \phi K$ have the same (dominant) internal quark-level process $b \to s\bar{s}s$, we take into account the possibility that there could be new physics effects on the process $b \to s\bar{s}s$: for instance, as considered in [18, 29] in order to explain the large negative value of $\sin(2\phi_1)_{\phi K_s}$ reported by Belle. We investigate whether it is possible to understand the difference between the BRs given in (22) and the experimental data, by invoking new physics.

As specific examples, we consider two new physics scenarios: R-parity violating (RPV) SUSY and R-parity conserving (RPC) SUSY.

(a) R-parity violating SUSY case

The RPV part of the superpotential of the minimal supersymmetric standard model can contain terms of the form

$$\begin{aligned} \mathcal{W}_{\rm RPV} = \kappa_i L_i H_2 + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c \\ + \lambda''_{ijk} U_i^c D_j^c D_k^c \,, \end{aligned} \tag{23}$$

where E_i, U_i and D_i are respectively the ith type of lepton, up-quark and down-quark singlet superfields, L_i and Q_i are the ${\rm SU(2)_L}$ doublet lepton and quark superfields, and H_2 is the Higgs doublet with the appropriate hypercharge.

For our purpose, we will assume only λ'-type couplings to be present. Then, the effective Hamiltonian for charmless hadronic B decay can be written as [16],

$$\begin{aligned} H_{\rm eff}^{\lambda'}(b \to \bar{d}_j d_k d_n) &= d_{jkn}^{\rm R} \left[\bar{d}_{n\alpha} \gamma_{\rm L}^\mu d_{j\beta} \;\; \bar{d}_{k\beta} \gamma_{\mu R} b_\alpha\right] \\ &\quad + d_{jkn}^{\rm L} \left[\bar{d}_{n\alpha} \gamma_{\rm L}^\mu b_\beta \;\; \bar{d}_{k\beta} \gamma_{\mu R} d_{j\alpha}\right], \\ H_{\rm eff}^{\lambda'}(b \to \bar{u}_j u_k d_n) &= u_{jkn}^{\rm R} \left[\bar{u}_{k\alpha} \gamma_{\rm L}^\mu u_{j\beta} \;\; \bar{d}_{n\beta} \gamma_{\mu R} b_\alpha\right]. \end{aligned} \tag{24}$$

Here the coefficients $d_{jkn}^{\rm L,R}$ and $u_{jkn}^{\rm R}$ are defined as

$$\begin{aligned} d_{jkn}^{\rm R} &= \sum_{i=1}^{3} \frac{\lambda'_{ijk} \lambda'^*_{in3}}{8 m^2_{\tilde{\nu}_{iL}}}, \\ d_{jkn}^{\rm L} &= \sum_{i=1}^{3} \frac{\lambda'_{i3k} \lambda'^*_{inj}}{8 m^2_{\tilde{\nu}_{iL}}} \quad (j,k,n = 1,2), \\ u_{jkn}^{\rm R} &= \sum_{i=1}^{3} \frac{\lambda'_{ijn} \lambda'^*_{ik3}}{8 m^2_{\tilde{e}_{iL}}} \quad (j,k = 1,\; n = 2), \end{aligned} \tag{25}$$

where α and β are color indices and $\gamma^\mu_{\rm R,L} \equiv \gamma^\mu (1 \pm \gamma_5)$. The leading order QCD correction to this operator is given by a scaling factor $f \simeq 2$ for $m_{\tilde{\nu}} = 200\,{\rm GeV}$. We refer to [16, 17] for the relevant notations.

The RPV SUSY part (relevant to the quark-level process $b \to s\bar{s}s$) of the decay amplitude of $B^- \to \eta' K^-$ is given by

$$\begin{aligned} \mathcal{A}^{\rm RPV}_{\eta' K} &= \left(d^{\rm L}_{222} - d^{\rm R}_{222}\right) \\ &\quad \times \left[\frac{\bar{m}}{m_s}\left(A^s_{\eta'} - A^u_{\eta'}\right)\left(\tilde{a}_6 + \frac{f^u_{\eta'}}{f^s_{\eta'}} \tilde{a}'_6\right)\right. \end{aligned}$$

$$+A^s_{\eta'}(\tilde{a}_4 - \tilde{a}_5) + A^u_{\eta'}\tilde{a}_4\Bigg], \quad (26)$$

where

$$\bar{m} \equiv \frac{m^2_{\eta'}}{(m_b - m_s)},$$
$$A^{u(s)}_{\eta'} = f^{u(s)}_{\eta'} F^{B\to K}(m_B^2 - m_K^2). \quad (27)$$

Here the coefficients $\tilde{a}_i^{(\prime)}$ are expressed as

$$\tilde{a}_4 = \frac{C_F\alpha_s}{4\pi N_c}\left[\frac{4}{3}\ln\frac{m_b}{\mu} - G_K(0)\right],$$
$$\tilde{a}_5 = \frac{1}{N_c}\left[1 - \frac{C_F\alpha_s}{4\pi}\left(V_{\eta'} + 12 + \frac{4\pi^2}{N_c}H(BK,\eta')\right)\right],$$
$$\tilde{a}_6 = 1 + \frac{C_F\alpha_s}{4\pi N_c}\left[\frac{4}{3}\ln\frac{m_b}{\mu} - \hat{G}_K(0)\right],$$
$$\tilde{a}'_6 = \frac{C_F\alpha_s}{4\pi N_c}\left[\frac{4}{3}\ln\frac{m_b}{\mu} - \hat{G}_K(0)\right], \quad (28)$$

where $G_K(0) = \frac{5}{3} + \frac{2\pi}{3}\mathrm{i}$ and $\hat{G}_K(0) = \frac{16}{9} + \frac{2\pi}{3}\mathrm{i}$.

It has been noticed [18] that $\mathcal{A}^{\mathrm{RPV}}_{\eta' K}$ is proportional to $(d^{\mathrm{L}}_{222} - d^{\mathrm{R}}_{222})$, while the RPV part of the decay amplitude of $B \to \phi K$ is proportional to $(d^{\mathrm{L}}_{222} + d^{\mathrm{R}}_{222})$. It has been also pointed out [18] that the opposite relative sign between d^{L}_{222} and d^{R}_{222} in the modes $B \to \eta' K$ and $B \to \phi K$ appears due to the different parity in the final state mesons η' and ϕ, and this different combination of $(d^{\mathrm{L}}_{222} - d^{\mathrm{R}}_{222})$ and $(d^{\mathrm{L}}_{222} + d^{\mathrm{R}}_{222})$ in these modes plays an important role to explain both the large BRs for $B \to \eta' K$ and the large negative value of $\sin(2\phi_1)_{\phi K_s}$ at the same time.

We define the new coupling terms d^{L}_{222} and d^{R}_{222} as follows:

$$d^{\mathrm{L}}_{222} \propto |\lambda'_{i32}\lambda'^*_{i22}|\mathrm{e}^{\mathrm{i}\theta_{\mathrm{L}}}\ ,\quad d^{\mathrm{R}}_{222} \propto |\lambda'_{i22}\lambda'^*_{i23}|\mathrm{e}^{\mathrm{i}\theta_{\mathrm{R}}}, \quad (29)$$

where θ_{L} and θ_{R} denote new weak phases of the product of new couplings $\lambda'_{i32}\lambda'^*_{i22}$ and $\lambda'_{i22}\lambda'^*_{i23}$, respectively, as defined by $\lambda'_{332}\lambda'^*_{322} \equiv |\lambda'_{332}\lambda'^*_{322}|\mathrm{e}^{\mathrm{i}\theta_{\mathrm{L}}}$ and $\lambda'_{322}\lambda'^*_{323} \equiv |\lambda'_{322}\lambda'^*_{323}|\mathrm{e}^{\mathrm{i}\theta_{\mathrm{R}}}$. We find that the experimental measurements of the BRs for $B^{+(0)} \to \eta' K^{+(0)}$ and $B^{+(0)} \to \phi K^{+(0)}$ can be consistently understood for the following values of the parameters:

$$|\lambda'_{322}| = 0.076\ ,\ |\lambda'_{332}| = 0.076\ ,\ |\lambda'_{323}| = 0.064\ ,$$
$$\theta_{\mathrm{L}} = 1.32\ ,\ \theta_{\mathrm{R}} = -1.29\ ,\ \ m_{\mathrm{SUSY}} = 200\,\mathrm{GeV}. \quad (30)$$

Our results are summarized in Table 3. In addition to the parameters given in (30), we also used the additional strong phase $\delta' = 30°$, which can arise from the power contributions of $\Lambda_{\mathrm{QCD}}/m_b$ [i.e., $\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) \sim |\mathrm{amplitude}|\mathrm{e}^{\mathrm{i}\delta'}$ in (4)] neglected in the QCDF scheme, and whose size can be in principle comparable to the strong phase arising from the radiative corrections of $O(\alpha_s)$. It has been shown [29] that using $\delta' = 30°$ together with the parameters given in (30), one can explain the large negative value of $\sin(2\phi_1)_{\phi K_s}$ as well. Notice that the new coupling terms d^{L}_{222} and d^{R}_{222} are relevant only to the process $b \to s\bar{s}s$, so they do not affect other $B \to PP$ and VP decays, such as $B \to \pi\pi$, πK, $\rho\pi$, ρK, et cetera, which are already well understood within the SM.

Table 3. The branching ratios (in unit of 10^{-6}) for $B \to \eta' K$ and $B \to \phi K$ decays calculated in the framework of R-parity violating SUSY. Here the inputs for the fit with small X_{A} and X_{H} are used

Decay mode	Prediction	Decay mode	Prediction
$B^+ \to \eta' K^+$	74.0	$B^0 \to \eta' K^0$	67.7
$B^+ \to \phi K^+$	10.2	$B^0 \to \phi K^0$	9.5

In the case of the large $X_{\mathrm{A,H}}$ effects with $\chi^2_{\mathrm{min}} = 7.5$ where the BR of $B^+ \to \eta' K^+$ is large, we can use the R-parity violating SUSY couplings to raise the BR of $B^+ \to \phi K^+$ (which is small, 4.02×10^{-6} to begin with). It is possible to raise $\mathcal{B}(B^+ \to \phi K^+)$ to $(8–9) \times 10^{-6}$. However, in this case, $\sin(2\phi_1)_{\phi K_s}$ cannot be large negative [29].

The RPV terms can arise in the context of SO(10) models which explain the small neutrino mass and has an intermediate breaking scale where $B - L$ symmetry gets broken by $(16 + \overline{16})$ Higgs. These additional Higgs form operators like $16_{\mathrm{H}}16_m16_m16_m/M_{pl}$ (16_m contains matter fields) and generate the RPV terms [30].

(b) R-parity conserving SUSY case

As an example of the RPC SUSY case, we will consider the supergravity (SUGRA) model with the simplest possible non-universal soft terms which is the simplest extension of the minimal SUGRA (mSUGRA) model. In this model the lightest SUSY particle is stable and this particle can explain the dark matter content of the universe. The recent WMAP result provides us with [31]

$$\Omega_{\mathrm{CDM}}h^2 = 0.1126^{+0.008}_{-0.009}, \quad (31)$$

and we implement a 2σ bound in our calculation.

In the SUGRA model, the superpotential and soft SUSY breaking terms at the grand unified theory (GUT) scale are given by

$$\mathcal{W} = Y^U QH_2U + Y^D QH_1D + Y^{\mathrm{L}} LH_1E + \mu H_1H_2,$$
$$\mathcal{L}_{\mathrm{soft}} = -\sum_i m_i^2|\phi_i|^2 - \Bigg[\frac{1}{2}\sum_\alpha m_\alpha\bar{\lambda}_\alpha\lambda_\alpha + B\mu H_1H_2 + (A^U QH_2U + A^D QH_1D + A^{\mathrm{L}} LH_1E) + \mathrm{H.c.}\Bigg], \quad (32)$$

where E, U and D are respectively the lepton, up-quark and down-quark singlet superfields, L and Q are the $\mathrm{SU(2)_L}$ doublet lepton and quark superfields, and $H_{1,2}$ are the Higgs doublets. ϕ_i and λ_α denote all the scalar fields and gaugino fields, respectively. In the mSUGRA model, a universal scalar mass m_0, a universal gaugino mass $m_{1/2}$, and the universal trilinear coupling A terms are introduced at the GUT scale:

$$m_i^2 = m_0^2, \quad m_\alpha = m_{1/2}, \quad A^{U,D,L} = A_0 Y^{U,D,L}, \quad (33)$$

where $Y^{U,D,L}$ are the diagonalized 3×3 Yukawa matrices. In this model, there are four free parameters, m_0, $m_{1/2}$, A_0, and $\tan\beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$, in addition to the sign of μ. The parameters $m_{1/2}$, μ and A can be complex, and four phases appear: θ_A (from A_0), θ_1 (from the gaugino mass m_1), θ_3 (from the gaugino mass m_3), and θ_μ (from the μ term).

It has been shown in [32,33] that the mSUGRA model cannot explain the large negative value of $\sin(2\phi_1)_{\phi K_s}$, because in this model the only source of flavor violation is in the CKM matrix, which cannot provide a sufficient amount of flavor violation needed for the $b \to s$ transition in the processes $B \to \phi K$. The minimal extension of the mSUGRA has been studied to solve the large negative $\sin(2\phi_1)_{\phi K_s}$ in the context of QCDF [32], or both large negative $\sin(2\phi_1)_{\phi K_s}$ and large BR of $B \to \eta' K$ in the context of NF [33].

The minimal extension of the mSUGRA model contains non-universal soft breaking A terms, in addition to the parameters in the mSUGRA model. In order to enhance contributions to the $b \to s$ transition, the simplest choice is to consider only non-zero (2,3) elements in A terms which enhance the left–right mixing of the second and third generation. The A terms with only non-zero (2,3) elements can be expressed as

$$A^{U,D} = A_0 Y^{U,D} + \Delta A^{U,D}, \quad (34)$$

where $\Delta A^{U,D}$ are 3×3 complex matrices and $\Delta A_{ij}^{U,D} = \left|\Delta A_{ij}^{U,D}\right| \mathrm{e}^{\mathrm{i}\phi_{ij}^{U,D}}$ with $\left|\Delta A_{ij}^{U,D}\right| = 0$ unless $(i,j) = (2,3)$ or $(3,2)$. It is obvious that the mSUGRA model is recovered if $\Delta A^{U,D} = 0$.

For our analysis, we consider all the known experimental constraints on the parameter space of the model, as in [32]. Those constraints come from the radiative B decay process $B \to X_s\gamma$ ($2.2 \times 10^{-4} < \mathcal{B}(B \to X_s\gamma) < 4.5 \times 10^{-4}$ [34,35]), neutron and electron electric dipole moments ($d_n < 6.3 \times 10^{-26} e\,\mathrm{cm}$, $d_e < 0.21 \times 10^{-26} e\,\mathrm{cm}$ [36]), relic density measurements, K^0–$\bar{K}^0$ mixing ($\Delta M_K = (3.490 \pm 0.006) \times 10^{-12}\,\mathrm{MeV}$ [36]), LEP bounds on masses of SUSY particles and the lightest Higgs ($m_h \geq 114\,\mathrm{GeV}$). From the experimental constraints, we find that $\theta_1 \approx 22°$, $\theta_3 \approx 30°$, and $\theta_\mu \approx -11°$. For the phase θ_A, we set $\theta_\mathrm{A} = \pi$. In general, the new physics contribution change the behavior of the annihilation contributions when the relevant Wilson coefficients are affected. It has been noticed [32] that the SUSY contribution in our case with non-zero ΔA_{23} terms mainly affects the Wilson coefficients $C_{8g(7\gamma)}$ and $\tilde{C}_{8g(7\gamma)}$ and these coefficients do not change the SM weak annihilation effects which depend on $C_{2,3,5-9}$ as in (12) (see [26] for calculational details of the weak annihilation).

In our calculation, we consider the case with non-zero ΔA_{23}^D and non-zero ΔA_{32}^D for $\tan\beta = 10$. All the other elements in $\Delta A^{U,D}$ are set to be zero. We compute the BRs for $B \to \eta' K$ and $B \to \phi K$ in the case of $\left|\Delta A_{23}^D\right| \sim \left|\Delta A_{32}^D\right|$ and $\phi_{23}^D \neq \phi_{32}^D$ with $\tan\beta = 10$. Table 4 shows the BRs for $B^+ \to \eta' K^+$ and $B^+ \to \phi K^+$ calculated for various values of the parameters $m_{1/2}$, $|A_0|$ and $|\Delta A_{23(32)}^D|$. For each $m_{1/2}$ and $|A_0|$, the left column shows the BR for $B^+ \to \eta' K^+$ and the right column shows the BR for $B^+ \to \phi K^+$. All the predicted BRs in the table are well consistent with the experimental data. The BR for $B^+ \to \eta K^+$ is estimated as $(3.1 \sim 4.4) \times 10^{-6}$ which also agrees with the data. The higher $\tan\beta$ values are also allowed, but the allowed range of $m_{1/2}$ becomes smaller. We satisfy the relic density constraint using the stau–neutralino coannihilation channel [37].

For the numerical calculation, we used the QCD parameters given in (21) and the additional strong phase $\delta' = 0$. The value of $m_{1/2}$ varies from 300 GeV to 600 GeV, and the value of $|A_0|$ varies from 0 to 800 GeV. Even though the value of m_0 is not explicitly shown, it is chosen for different $m_{1/2}$ and A_0 such that the relic density constraint is satisfied, e.g., for $m_{1/2} = 300\,\mathrm{GeV}$, m_0 varies in the range (70–110) GeV. The value of m_0 increases as $m_{1/2}$ increases. The value of $\left|\Delta A_{23(32)}^D\right|$ increases as $m_{1/2}$ does. The phases ϕ_{23}^D and ϕ_{32}^D are approximately $-40°$ to $-15°$ and $165°$ to $180°$, respectively. So far we have assumed that $\Delta A_{23,32}^U = 0$. But if we use $\Delta A_{23,32}^U \neq 0$ and $\Delta A_{23,32}^D = 0$, the value of $\sin(2\phi)_{\phi K_s}$ is mostly positive.

In passing, we note that the set of the same parameters used in our calculation can also produce a large negative value of $\sin(2\phi_1)_{\phi K_s}$ [29]. As a final comment, we note that in the case of the large $X_{\mathrm{A,H}}$ effects with $\chi^2_{\min} = 7.5$, it is possible to raise the BR for $B^+ \to \phi K^+$ to $(8–9) \times 10^{-6}$. However, in that case, the large negative value of $\sin(2\phi_1)_{\phi K_s}$ cannot be obtained [29].

Table 4. The branching ratios (in units of 10^{-6}) for $B^+ \to \eta' K^+$ (left) and $B^+ \to \phi K^+$ (right) at $\tan\beta = 10$ with non-zero ΔA_{23}^D and ΔA_{32}^D. The units for $m_{1/2}$, $|A_0|$, and $\left|\Delta A_{23(32)}^D\right|$ are in GeV

$\lvert A_0\rvert$	800		600		400		0		$\lvert\Delta A_{23(32)}^D\rvert$
$m_{1/2} = 300$	79.6	9.9	81.0	9.2	79.6	9.1	79.0	8.1	66–74
$m_{1/2} = 400$	78.2	9.9	83.0	9.6	79.0	9.2	81.0	8.5	150–168
$m_{1/2} = 500$	84.8	9.9	83.7	9.9	81.0	10.0	77.0	8.1	244–256
$m_{1/2} = 600$	73.0	7.6	71.0	7.5	70.0	7.5	70.0	7.1	270–304

5 Conclusion

We investigated the decay processes $B^{+(0)} \to \eta' K^{+(0)}$ in the QCDF approach. In order to reliably estimate the weak annihilation parameter $X_{\rm A} \equiv \left(1+\rho_{\rm A} e^{i\phi_{\rm A}}\right) \ln\frac{m_B}{\Lambda_h}$ and the hard spectator scattering parameter $X_{\rm H} \equiv \left(1+\rho_{\rm H} e^{i\phi_{\rm H}}\right) \ln\frac{m_B}{\Lambda_h}$ arising from logarithmic divergences in the end-point region, we used the global analysis for twelve $B \to PP$ and VP decay modes, such as $B \to \pi\pi$, πK, $\rho\pi$, ρK, $\omega\pi$, ωK. From the global analysis, we found that both the large effect of $X_{\rm A,H}$ (less reliable) and the small effect of $X_{\rm A,H}$ (more reliable) are allowed. For the former case, the parameters $\rho_{\rm A,H}$ and $\phi_{\rm A,H}$ are determined to be $\rho_{\rm A}^{PP} = \rho_{\rm A}^{VP} = \rho_{\rm H}^{PP} = \rho_{\rm H}^{VP} = 1$, $\phi_{\rm A}^{PP} = 57°$, $\phi_{\rm A}^{VP} = 52°$, $\phi_{\rm H}^{PP} = -23°$, $\phi_{\rm H}^{VP} = 180°$. For the latter case, the parameters $\rho_{\rm A,H}$ and $\phi_{\rm A,H}$ are $\rho_{\rm A}^{PP} = 0$, $\rho_{\rm A}^{VP} = 0.5$, $\rho_{\rm H}^{PP} = 1$, $\rho_{\rm H}^{VP} = 0.746$, $\phi_{\rm A}^{VP} = -6°$, $\phi_{\rm H}^{VP} = \phi_{\rm H}^{PP} = 180°$.

In the case of the large $X_{\rm H,A}$ effects allowed by the "good" fit (with $\chi^2_{\rm min} = 8.6$ for the twelve decay modes), the BRs for $B^{+(0)} \to \eta' K^{+(0)}$ and $B^{+(0)} \to \phi K^{+(0)}$ calculated within the SM saturate the large values of the experimental results measured by Belle, BaBar, and CLEO. Thus, there is no room for invoking new physics effects on the quark-level process $b \to s\bar{s}s$, which are implied by the large negative value of $\sin(2\phi_1)_{\phi K_s}$ recently reported by Belle.

In contrast, in the case of the small $X_{\rm H,A}$ effects that is theoretically more reliable, the SM prediction for these BRs is smaller than the experimental data. Since both $B^{+(0)} \to \eta' K^{+(0)}$ and $B^{+(0)} \to \phi K^{+(0)}$ have the same (dominant) internal process $b \to s\bar{s}s$, we took into account possible new physics effects on the $b \to s\bar{s}s$ transition, as in [18,29] for explaining the recent Belle measurement of $\sin(2\phi_1)_{\phi K_s}$. Specifically, we considered two new physics scenarios: R-parity violating SUSY and R-parity conserving SUSY. In the RPV SUSY case, the BRs for $B^{+(0)} \to \eta' K^{+(0)}$ are predicted as $73.9(67.8) \times 10^{-6}$ and the BRs for $B^{+(0)} \to \phi K^{+(0)}$ are $10.2(9.5) \times 10^{-6}$ which are consistent with the data. The relevant new couplings are found to be $|\lambda'_{322}| = 0.086$, $|\lambda'_{332}| = 0.089$, $|\lambda'_{323}| = 0.030$, $\theta_{\rm L} = 0.66$, $\theta_{\rm R} = -2.25$. As an example of the RPC SUSY case, we adopted the simplest extension of the mSUGRA model, which contains only non-zero (2,3) elements in the soft breaking trilinear coupling A terms, in addition to the other parameters of the mSUGRA model. Considering all the known constraints on the relevant parameter space, we found that for $\tan\beta = 10$, $\mathcal{B}(B^+ \to \eta' K^+) = (70.0–84.8) \times 10^{-6}$ and $\mathcal{B}(B^+ \to \phi K^+) = (7.1–10.0) \times 10^{-6}$, which are in good agreement with the experimental data.

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